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Fill in a(t) and b(t) in (4.38) with the expressions given in (4.40):

$$\frac{\hbar}{2}\begin{pmatrix}\omega_0 & \omega_1\cos\omega t\\ \omega_1\cos\omega t & -\omega_0\end{pmatrix}\begin{pmatrix} c(t)e^{-i\omega_0 t/2}\\ d(t)e^{i\omega_0 t/2}\end{pmatrix} = i\hbar\begin{pmatrix}\frac{d}{dt}(c(t)e^{-i\omega_0 t/2})\\ \frac{d}{dt}(d(t)e^{i\omega_0 t/2})\end{pmatrix}$$

Dividing $\hbar$ from both sides and multiplying out the matrix on the left:

$$\frac{1}{2}\begin{pmatrix}\omega_0 c(t)e^{-i\omega_0 t/2}+\omega_1\cos{(\omega t)}d(t)e^{i\omega_0 t/2}\\ \omega_1\cos{(\omega t)}c(t)e^{-i\omega_0 t/2}-\omega_0 d(t)e^{i\omega_0 t/2}\end{pmatrix} = i\begin{pmatrix}\frac{d}{dt}(c(t)e^{-i\omega_0 t/2})\\ \frac{d}{dt}(d(t)e^{i\omega_0 t/2})\end{pmatrix}$$

Taking the derivative of the right side using the product rule:

$$\frac{1}{2}\begin{pmatrix}\omega_0 c(t)e^{-i\omega_0 t/2}+\omega_1\cos{(\omega t)}d(t)e^{i\omega_0 t/2}\\ \omega_1\cos{(\omega t)}c(t)e^{-i\omega_0 t/2}-\omega_0 d(t)e^{i\omega_0 t/2}\end{pmatrix} = i\begin{pmatrix}\frac{dc}{dt}e^{-i\omega_0 t/2}+c(t)(\frac{-i\omega_0}{2})(e^{-i\omega_0 t/2})\\ \frac{dd}{dt}e^{i\omega_0 t/2}+d(t)(\frac{i\omega_0}{2})(e^{i\omega_0 t/2})\end{pmatrix}$$

$$\frac{1}{2}\begin{pmatrix}\omega_0 c(t)e^{-i\omega_0 t/2}+\omega_1\cos{(\omega t)}d(t)e^{i\omega_0 t/2}\\ \omega_1\cos{(\omega t)}c(t)e^{-i\omega_0 t/2}-\omega_0 d(t)e^{i\omega_0 t/2}\end{pmatrix} = i\begin{pmatrix}\frac{dc}{dt}e^{-i\omega_0 t/2}\\ \frac{dd}{dt}e^{i\omega_0 t/2}\end{pmatrix}+i\begin{pmatrix}c(t)(\frac{-i\omega_0}{2})(e^{-i\omega_0 t/2})\\ d(t)(\frac{i\omega_0}{2})(e^{i\omega_0 t/2})\end{pmatrix}$$

$$\frac{1}{2}\begin{pmatrix}\omega_0 c(t)e^{-i\omega_0 t/2}+\omega_1\cos{(\omega t)}d(t)e^{i\omega_0 t/2}\\ \omega_1\cos{(\omega t)}c(t)e^{-i\omega_0 t/2}-\omega_0 d(t)e^{i\omega_0 t/2}\end{pmatrix} = i\begin{pmatrix}\frac{dc}{dt}e^{-i\omega_0 t/2}\\ \frac{dd}{dt}e^{i\omega_0 t/2}\end{pmatrix}+\frac{\omega_0}{2}\begin{pmatrix}c(t)(e^{-i\omega_0 t/2})\\ -d(t)(e^{i\omega_0 t/2})\end{pmatrix}$$

Subtracting the matrix on the far right on both sides:

$$\frac{1}{2}\begin{pmatrix}\omega_1\cos{(\omega t)}d(t)e^{i\omega_0 t/2}\\ \omega_1\cos{(\omega t)}c(t)e^{-i\omega_0 t/2}\end{pmatrix} = i\begin{pmatrix}\frac{dc}{dt}e^{-i\omega_0 t/2}\\ \frac{dd}{dt}e^{i\omega_0 t/2}\end{pmatrix}$$

$$\frac{\omega_1\cos{(\omega t)}}{2}\begin{pmatrix}d(t)e^{i\omega_0 t/2}\\ c(t)e^{-i\omega_0 t/2}\end{pmatrix} = i\begin{pmatrix}\frac{dc}{dt}e^{-i\omega_0 t/2}\\ \frac{dd}{dt}e^{i\omega_0 t/2}\end{pmatrix}$$

Now the right side of the equation can be rewritten as a matrix times a column vector:

$$\frac{\omega_1\cos{(\omega t)}}{2}\begin{pmatrix}d(t)e^{i\omega_0 t/2}\\ c(t)e^{-i\omega_0 t/2}\end{pmatrix} = i\begin{pmatrix}e^{-i\omega_0 t/2} & 0\\ 0 & e^{i\omega_0 t/2}\end{pmatrix}\begin{pmatrix}\frac{dc}{dt}\\ \frac{dd}{dt}\end{pmatrix}$$

Taking the inverse of the matrix on both sides:

$$\frac{\omega_1 \cos(\omega t)}{2} \begin{pmatrix} e^{i\omega_0 t/2} & 0 \\ 0 & e^{-i\omega_0 t/2} \end{pmatrix} \begin{pmatrix} d(t)e^{i\omega_0 t/2} \\ c(t)e^{-i\omega_0 t/2} \end{pmatrix} = i \begin{pmatrix} \frac{dc}{dt} \\ \frac{dd}{dt} \end{pmatrix}$$

$$\frac{\omega_1 \cos(\omega t)}{2} \begin{pmatrix} d(t)e^{i\omega_0 t} \\ c(t)e^{-i\omega_0 t} \end{pmatrix} = i \begin{pmatrix} \frac{dc}{dt} \\ \frac{dd}{dt} \end{pmatrix}$$